Connections between different fields of deductive reasoning in the thinking of 13 and 17 year olds

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ABSTRACT

This study focuses on different fields of deductive reasoning: basic operations of propositional logic, inference rules and Wason selection task.

The Wason subtest consisted of 9 tasks, each with its 4 items according to the possible cases of the presence of 0-2 verification and 0-2 falsification cards. The subtest of the inference rules diagnosed the status of modus ponens, affirmation of consequent, denial of antecedent and modus tollens.

The results show:
(1) the modus tollens inference rule is not considered as the clue to the falsification principle;
(2) the histogram of students’ achievement in the Wason subtest has local maximum values which indicates the existence of relatively consistent reasoning structures. This consistency yielded relatively better results in the tasks of modus ponens and modus tollens;
(3) the attainment of the modus ponens inference rule is of great importance in good performance in other tasks;
(4) the attainment of the conditional operation involves higher performance in the inference rules;
(5) the nature of the connections between different fields of deductive reasoning are similar for the two age groups.

The data collection was carried out in a project supported by the Hungarian National Science Foundation (OTKA TO18577)
Aims

According to former investigations there are deductive reasoning tasks with great difficulties for both adults and children. These tasks require the use of metalogical strategies or appreciating the falsification principle. Wason selection task, propositional logic, inference rules etc. have their own bulky bibliography, but only a few investigations have dealt with connections between these fields of deductive reasoning.

Considering that the solution of certain difficult tasks may require the coordination of relatively simple inference rules, the aims of the present investigation were:

(1) to construct a test system for the evaluation of basic operations of propositional logic, determinate and plausible inference rules and the Wason selection task
(2) to diagnose the status of these operations, inference rules and the Wason selection task in the age groups of 13 and 17 year olds
(3) to reveal connections between the three different fields of deductive reasoning
(4) to reveal significant differences between the two age groups

Methods and tasks

Three subtests were constructed for (1) Wason selection task, (2) the 7 basic operations of propositional logic, (3) two determinate and two plausible inference rules.

Two types of the Wason task system were constructed with different content. The first version consisted of cards with the ‘usual’ content: letters and numbers. The second version consisted of cards with personal names and names of towns, without obvious relation between them. The Wason subtest consisted of 9 tasks, each with its 4 items according to the possible cases of the presence of 0–2 verification and 0–2 falsification cards. (With the rule to be tested ‘If p, then q’ there were two cards in each task to represent p or not-p, and two cards to represent q or not-q. The first task consisted of two p and two q cards, the second one consisted of one p, one not-p and two q cards etc. We can call a p card as a verification card, and a not-q card as a falsification card.)

This task system contained the classical selection task with p, not-p, q, and not-q cards. What is more, with these 9 tasks we can examine the consistency that may appear while solving Wason’s selection task. The third subtest consisted of four tasks, each diagnosing the status of modus ponens, affirmation of consequent, denial of antecedent and modus tollens.

Striving for presenting content free inference rule tasks, the premises contained statements about sports and weather, avoiding obvious relation between the antecedent and the consequent.

The test was administered to two age groups, 515 13 year old and 488 17 year old students in Szeged.

The reliability parameters were appropriate. The Cronbach–α was higher than 0.74 for each subtest and age group. The best parameter was 0.93 for the subtest of basic operations of the 13 year olds.

Statistical methods that were used: descriptive statistics, correlations, t-tests, regression analysis.
Results

Wason selection tasks proved to be difficult for the two age groups. This task system consisted of 36 items, and the histogram of students' achievement showed local maximum values at 0, 9, 18, 27 and 36 points. (See Figure.) This indicates the existence of relatively consistent reasoning structures, thus contributes to the well-known 'insight'-models. Through the Wason task system the most important result was that this consistency in thinking yielded relatively better results in the tasks of modus ponens and modus tollens. (See Table 1.) The hypothesis 'Modus tollens guides the strategy of falsification' was tested with the help of Wason and inference rules subtests by way of different methods. Comparing group means by cut point 18 in Wason task system there was no significant difference in performance in modus tollens items. (See Table 2.) On the other hand, high performance in modus tollens items does not involve high performance in falsification card items. These results indicate that the modus tollens inference rule is not considered as the clue to the falsification principle.

Close connections can be demonstrated between attaining implication and results in inference rules especially in the denial of antecedent plausible inference rule. According to the results of the present investigation it is clear that acquiring modus ponens is of capital importance in deductive reasoning. Children who do not attain this inference rule produce significantly weaker results in other tasks as well.

The performance on almost every tasks shows significant differences between the two age groups, but the nature of the connections between different fields of deductive reasoning is about the same with the exception of connection between falsification cards' items and inference rules with different grammatical forms of the major premise. At the age of 13, significant positive correlations exist between falsification card items and 'If not p, then q' forms. At the age of 17, the form 'If p, then not q' shows positive correlations.

Table 1.

Results in modus ponens and modus tollens tasks (0-4 points) according to the seven groups by results on the Wason subtest

<table>
<thead>
<tr>
<th></th>
<th>Modus ponens</th>
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<tbody>
<tr>
<td>0-7</td>
<td>2.66</td>
<td>1.91</td>
</tr>
<tr>
<td>8-10</td>
<td>2.85</td>
<td>2.29</td>
</tr>
<tr>
<td>11-17</td>
<td>2.80</td>
<td>2.00</td>
</tr>
<tr>
<td>18</td>
<td>3.57</td>
<td>2.82</td>
</tr>
<tr>
<td>19-25</td>
<td>2.64</td>
<td>2.23</td>
</tr>
<tr>
<td>26-28</td>
<td>3.28</td>
<td>2.62</td>
</tr>
<tr>
<td>29-36</td>
<td>3.81</td>
<td>2.50</td>
</tr>
</tbody>
</table>

Table 2.

Results in modus tollens task (0-4 points) according to two groups by results on the Wason subtest

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<tr>
<td></td>
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(t=0.69; p<0.05)
Figure 1.

Histogram of scores on the Wason subtest (%).
Conclusion

The test system designed has proved to be an appropriate instrument for testing different fields of deductive patterns.
Contrary to previous expectations, the results do not indicate any simpler clue to the falsification principle.
Thus it appears likely that the Wason selection task requires not only deductive reasoning processes to solve.
The most important new finding is that consistency in solving Wason task system indicated better results in simpler tasks.

Special literature for the theoretical framework:


ABSTRACT

This study focuses on different fields of deductive reasoning: basic operations of propositional logic, inference rules and Wason selection task.

The Wason subtest consisted of 9 tasks, each with its 4 items according to the possible cases of the presence of 0-2 verification and 0-2 falsification cards. The subtest of the inference rules diagnosed the status of modus ponens, affirmation of consequent, denial of antecedent and modus tollens.

The results show:

(1) the modus tollens inference rule is not considered as the clue to the falsification principle;

(2) the histogram of students' achievement in the Wason subtest has local maximum values which indicates the existence of relatively consistent reasoning structures. This consistency yielded relatively better results in the tasks of modus ponens and modus tollens;

(3) the attainment of the modus ponens inference rule is of great importance in good performance in other tasks;

(4) the attainment of the conditional operation involves higher performance in the inference rules;

(5) the nature of the connections between different fields of deductive reasoning are similar for the two age groups.
Aims

Former investigations have revealed that there are deductive reasoning tasks with great difficulties for children and adults both. These tasks require the use of metalogical strategies or appreciating the falsification principle. The Wason selection task, propositional logic, inference rules etc. have their own bulky bibliography, but only a few investigations have dealt with the connections between these fields of deductive reasoning. Considering that the solution of certain difficult tasks may require the coordination of relatively simple inference rules, the aims of the present investigation were:

(1) to construct a test system for the evaluation of basic operations of propositional logic, determinate and plausible inference rules and the Wason selection task;
(2) to diagnose the status of these operations, inference rules and the Wason selection task in the age groups of 13 and 17 year olds;
(3) to reveal connections between the three different fields of deductive reasoning; to scrutinize hypotheses like ‘Modus tollens guides the strategy of falsification’, ‘Modus ponens is of great importance in deductive reasoning’ etc.;
(4) to reveal significant differences between the two age groups.
Methods and tasks (Part I)

Three subtests were constructed for (1) Wason selection task, (2) the 7 basic operations of propositional logic, (3) two determinate and two plausible inference rules.

Two types of the Wason task system were constructed with different content. The first version consisted of cards with the ‘usual’ letters and numbers. The second version consisted of cards with personal names and names of towns, without obvious relation between them. The Wason subtest consisted of 9 tasks, each with its 4 items according to the possible cases of the presence of 0–2 verification and 0–2 falsification cards.

(With the rule to be tested ‘If p, then q’ there were two cards in each task to represent p or not-p, and two cards to represent q or not-q. The first task consisted of two p and two q cards, the second one consisted of one p, one not-p and two q cards etc. We can call a p card as a verification card, and a not-q card as a falsification card.)

The task system contained the classical selection task with \( p, \, \text{not}-p, \, q, \) and \( \text{not}-q \) cards. What is more, with these 9 tasks we can examine the consistency that may appear while solving Wason’s selection task.
Methods and tasks *(Part II)*

The system of the tasks of basic operations was the same as described by Vidákovich (1996). The third subtest consisted of four tasks, diagnosing the status of modus ponens, affirmation of consequent, denial of antecedent and modus tollens.

Striving to present content free inference rule tasks, the premises contained statements about sports and weather, avoiding obvious relations between the antecedents and the consequents.

The test was administered to two age groups, 515 13 year old and 488 17 year old students in Szeged, Hungary.

The reliability parameters were appropriate. The Cronbach–α was higher than 0.74 for each subtest and age group.

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Results (Part I)

The Wason selection tasks proved to be difficult for both age groups. This task system consisted of 36 items, and the histogram of students' achievement showed local maximum values at 0, 9, 18, 27 and 36 points. (See Figure.) This indicates the existence of relatively consistent reasoning structures. The most important result was that this consistency in thinking seems to yield relatively better results in the tasks of modus ponens and modus tollens. (See Table 1.) The hypothesis 'Modus tollens guides the strategy of falsification' was tested with different methods, with the help of the Wason and inference rules subtests. Comparing group means by cut point 18 in the Wason task system, there was no significant difference found in performance on modus tollens items. (See Table 2.) On the other hand, a high performance in modus tollens items does not seem to indicate high performance in falsification card items.
Results (Part II)

These results suggest that the *modus tollens* inference rule is not considered as the clue to the *falsification principle*.

Close connections can be demonstrated between the attainment of implication and results in inference rules, especially in the denial of antecedent plausible inference rule.

The results of the present investigation suggest that the *acquisition of modus ponens is of capital importance in deductive reasoning*. Children who do not attain this inference rule produce significantly weaker results in other tasks as well.

The performance on almost every tasks shows significant differences between the two age groups, but the nature of the connections between different fields of deductive reasoning seems to be similar.
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Thus it appears likely that the Wason selection task requires not only deductive reasoning processes to solve.

The most important new finding is that consistency in solving the Wason task system indicated better results on simpler tasks.