```
> restart:with(plots):
Warning, the name changecoords has been redefined
```

Differenciálegyenlet $(\Delta - a^2)\,\Phi(x, y, z) = -4\,\pi\,\rho;\ \rho = \frac{e^{(-r)}}{8\,\pi}$

Gömbszimmetrikus megoldásra $\phi(r) = r\,\Phi(r),\ \left(\frac{\partial^2}{\partial r^2}\phi\right) - a^2\,\phi = -4\,\pi\,r\,\rho,\ \phi(0) = 0,\ \phi(\infty) = 0$

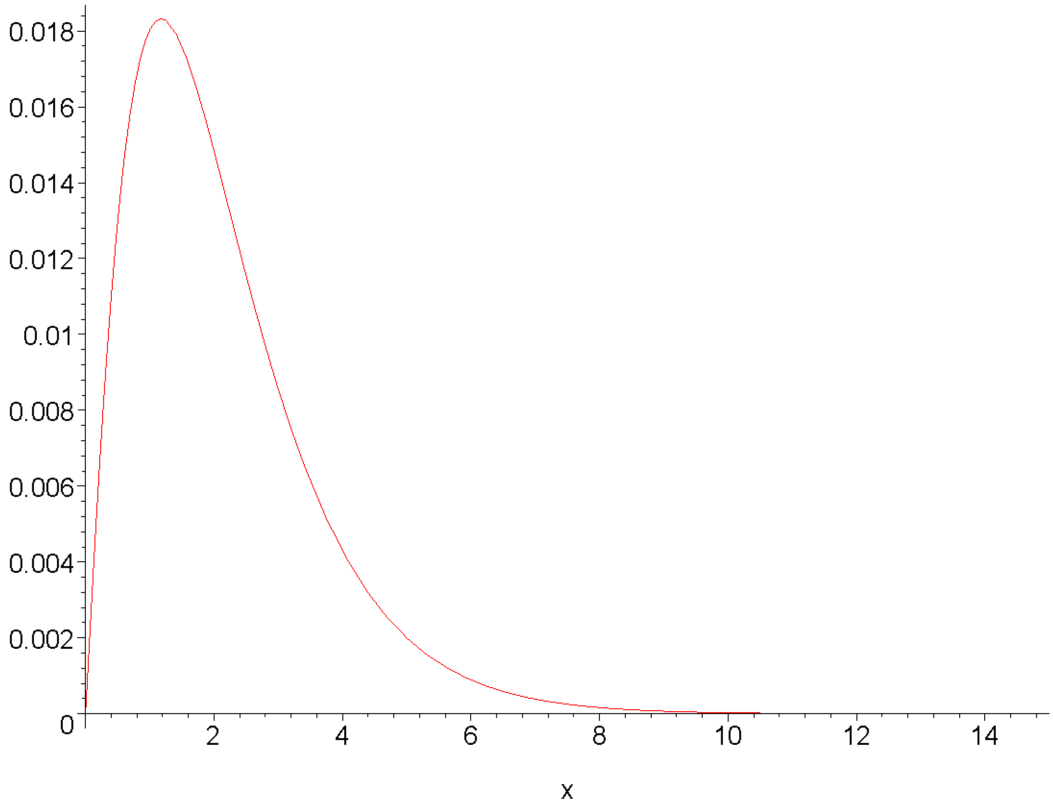
Diszkretizálva $\phi_{i+1} - (2 + a^2\,\delta^2)\,\phi_i + \phi_{i-1} = -\frac{\delta^2\,(r\,e^{(-r)})_i}{2}$

Az egzakt megoldás $\phi := r \to \frac{e^{(-a\,r)} - e^{(-r)}\left(1 + \frac{b\,r}{2}\right)}{b^2};\ b := 1 - a^2$

> simplify$\left(\left(\frac{\partial^2}{\partial r^2}\phi(r)\right) - a^2\,\phi(r) + \frac{r\,e^{(-r)}}{2}\right)$

$$0$$

```
> a:=3.:mx:=5*a:
  pl1:=plot(phi(x),x=0..mx):display(pl1);
```



A numerikus megoldás a tridiagonális egyenletrendszer elõre-hátra helyettesítésével

$mx := 15.;\ n := 160;\ \delta := \frac{mx}{n+1}$

$$\delta := .09316770186$$

A tridiagonális mátrix elkészítése

$\alpha := 1.; \beta := -(2+\delta^2 a^2); gama := 1.;$ **for** i **to** n **do** $x_i := i\,\delta; d_i := -\frac{\delta^2 x_i\, \mathbf{e}^{(-x_i)}}{2}$ **end do**

Hátra helyettesítés

$g_{n-1} := -\frac{\alpha}{\beta};$

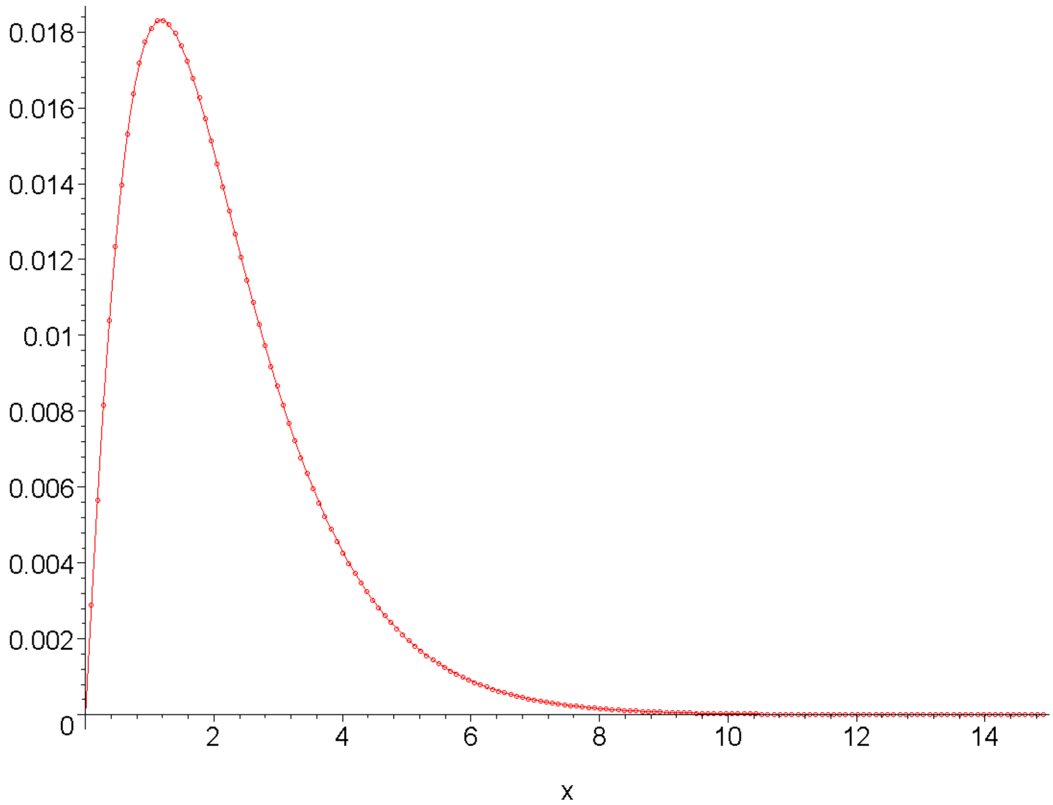
$h_{n-1} := \frac{d_n}{\beta};$

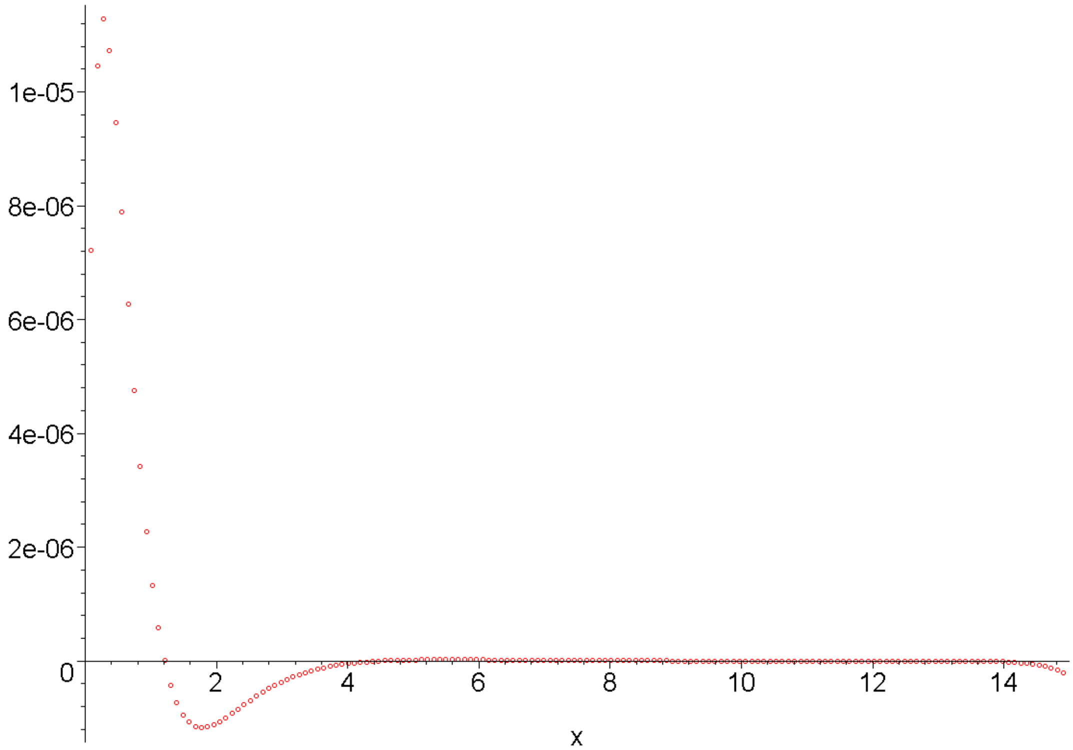
for i **from** $n-1$ **by** -1 **to** 2 **do** $g_{i-1} := -\frac{\alpha}{\beta + gama\, g_i}; h_{i-1} := \frac{d_i - gama\, h_i}{\beta + gama\, g_i}$ **end do**

Elõre helyettesítés

$\psi_1 := \frac{d_1 - gama\, h_1}{\beta + gama\, g_1};$ **for** i **to** $n-1$ **do** $\psi_{i+1} := g_i\, \psi_i + h_i$ **end do**

```
> pl1:=plot(phi(x),x=0..mx):
  l := [[ x[k], psi[k]] $k=1..n]:
  pl2:=plot(l, x=0..mx, style=point,symbol=circle):
> display({pl1,pl2});
> l := [[ x[k], psi[k]-phi(x[k])] $k=1..n]:
  plot(l, x=0..mx, style=point,symbol=circle);
```





1e-05
8e-06
6e-06
4e-06
2e-06
0
2
4
6
8
10
12
14
x

>

>