Introduction to mathematical expressions

with International Phonetic Alphabet (IPA)

https://en.wikipedia.org/wiki/International_Phonetic_Alphabet

1 Notation (noʊˈteɪ ʃən)

$${}^{a}_{b}x$$

where "a" is the left hand superscript (ˈsu pərˌskrɪpt), "b" is the left hand subscript (ˈsʌb skrɪpt)

$$x^{c}_{d}$$

where "c" is the right hand superscript, e.g., (e.g. means for example; for the sake of example; such as) power (ˈpaʊ ər), "d" is the right hand subscript, e.g., index (ˈɪn dɛks).

$$y = f(x)$$

In mathematics, a function is a rule for taking an input (in the simplest case, a number or set of numbers) and providing an output (which may also be a number). A symbol that stands for an arbitrary input is called an independent (ˌɪn dɪˈpɛn dənt) variable (ˈvɛər i ə bəl), while a symbol that stands for an arbitrary output is called a dependent (dependant (dɪˈpɛn dənt) variable. The most common symbol for the input is x, and the most common symbol for the output is y; the function itself is commonly written y=f(x).

2 Addition (əˈdɪʃ ən)

$$a + b = c$$

addends **sum**

(ˈæd ɛnd, əˈdɛnd) (sʌm)

To be read:

a plus b is equal (to, with) c (ai ˈplʌs ˈbiː ˈɪz ˈiːkwəl (ˈtuː wɪð) ˈsiː)

a plus b equals (to, with) c (ai ˈplʌs ˈbiː ˈiːkwəlz (ˈtuː wɪð) ˈsiː)

a plus b is c (ai ˈplʌs ˈbiː ˈɪz ˈsiː)

a added to b is c (equals to c, etc.)

the **sum** of a and b is c

3 Summation (səˈmeɪ ʃən)

$$\sum_{i=1}^{n} a_i = b$$

In mathematics, summation is the addition of a sequence of any kind of numbers, called addends or summands (ˈsʌm ænd, sʌmˈænd, səˈmænd); the result is their sum or total.

$$a_1 + a_2 + a_3 + \cdots + a_n = \sum_{i=1}^{n} a_i = b$$

Σ is an enlarged capital Greek letter sigma (ˈsɪg mə), but in this context its pronunciation is sum.

To be read: Sum from i equals 1 to n a i is b.
(ˈsʌm frəm ˈaɪ ˈiːkwəlz ˈwʌn ˈtuː ˈɛn eɪ ˈaɪ ˈɪz ˈbiː)

The subscript gives the symbol for a dummy variable (i in this case), called the "index of summation" together with its lower bound (1), whereas the superscript (here n) gives its upper bound. The lower and upper bound are expressions denoting integers. The factors of the sum are obtained by taking the expression following the summation operator, with successive integer values substituted for the index of addition, starting from the lower bound and incremented by 1 up to and including the upper bound. So, for example:

$$\sum_{i=1}^{5} i = 1 + 2 + 3 + 4 + 5 = 15$$

To be read: Sum from i equals 1 to n i is 15
ˈsʌm frəm ˈaɪ ˈiːkwəlz ˈwʌn ˈtuː ˈfaɪv ˈaɪ ˈɪz ˌfɪfˈtiːn

4 Subtraction (səbˈtræk ʃən)

$$\boldsymbol{a - b = c}$$

To be read:
a minus (ˈmaɪ nəs) b is equal (to, with) c
a minus b equals to c
a minus b is c
b subtracted from a is c (equals to c, etc.)
the **difference** (ˈdɪf ər əns, ˈdɪf rəns) of a and b is c

The change of a certain quantity (e.g., temperature, T) is often denoted as the difference from its initial value to its final value. It is expressed by a capital Greek letter delta (ˈdɛl tə)
$\Delta T = T_{final} - T_{initial}$
To be read: delta T

5 Multiplication (ˌmʌl tə plɪˈkeɪ ʃən)

$$\boldsymbol{a \times b = a \cdot b = a * b = c}$$

multiplicand	* multiplier	product
(ˌmʌl tə plɪˈkænd	ˈmʌl təˌplaɪ ər	ˈprɒd əkt, -ʌkt)

To be read:
a multiplied by b is equal to c
a multiplied by b equals to c
a multiplied by b is c
a times b is c (equals to c, etc.)
the **product** of a and b is c

6 Product of sequences

$$\prod_{i=1}^{n} a_i = b$$

The product of a sequence of terms can be written with the product symbol, which derives from the capital letter Π (Pi) in the Greek alphabet but in this context its pronunciation is product (ˈprɒd əkt, -ʌkt).

$$a_1 \cdot a_2 \cdot a_3 \cdot \cdots \cdot a_n = \prod_{i=1}^{n} a_i = b$$

The subscript gives the symbol for a dummy variable (i in this case), called the "index of multiplication" together with its lower bound (1), whereas the superscript (here n) gives its upper bound. The lower and upper bound are expressions denoting integers. The factors of the product are obtained by taking the expression following the product operator, with successive integer values substituted for the index of multiplication, starting from the lower bound and incremented by 1 up to and including the upper bound. So, for example:

$$\prod_{i=1}^{5} i = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$$

To be read: Product from i equals 1 to n i is 120

(ˈprɒdʌkt frəm ˈaɪ ˈiːkwəlz ˈwʌn ˈtuː ˈfaɪv ˈaɪ ˈɪz ˈhʌn drɪd ænd twɛlv)

7 Division (dɪˈvɪʒ ən)

$$a \div b = a : b = a/b = \frac{a}{b} = c$$

To be read:

a divided (dɪˈvaɪ dɪd) by b is equal to c

a divided by b equals to c

a divided by b is c

a by (baɪ) b is c (equals to c, etc.)

a over (ˈoʊ vər) b is c (equals to c, etc.)

A way to express division all on one line is to write the dividend (or numerator), then a slash (slæʃ) , then the divisor (or denominator), like this: a/b

$$a/b = \frac{a}{b} = \frac{dividend}{divisor} = \frac{numerator}{denominator} = quotient$$

(ˈdɪv ɪˌdɛnd dɪˈvaɪ zər ˈnu məˌreɪ tər, ˈnyu- dɪˈnɒm əˌneɪ tə ˈkwoʊ ʃənt)

The result of the division is called quotient, or a/b could be called as the ratio (ˈreɪ ʃoʊ, -ʃiˌoʊ) of a and b if they are simple numbers.

1/b is called the reciprocal (rɪˈsɪp rə kəl) of b.

The naming of rational fractions (ˈfræk ʃən) (quotients of whole numbers)

½ half

1/3 one third

¼ quarter

2/5 two fifth. etc.

i.e., the numerator is a normal number, the denominator is an ordinal number.

The naming of decimal fractions
0 (the number) is called zero (ˈzɪər oʊ) or nought (nɔt) or naught (nɔt) if something is perfectly nil (nɪl)
0 (the number) is called O (oh, oʊ) if it is part of a decimal number
E.g., a + b = 0, the sum of a and b is zero.
0.01 g of mass, to be read as oh point oh one gram or as one hundredth gram
0.0124 g of mass to be read as oh point oh one two four gram
With measurement units the division is often abbreviated by "per" (pɜr; unstressed pər) meaning "for each; for every"
E.g.,

$$mph = \frac{miles\ travelled}{hours\ needed} = miles\ for\ each\ hour = miles\ per\ hour = mph$$

In chemistry:

$$concentration = \frac{moles}{liters} = moles\ for\ each\ liter = moles\ per\ liter = \frac{mol}{L}$$

8 Exponentiation (ˌɛk spoʊˌnɛn ʃiˈeɪ ʃən, -spə-)

$$\boldsymbol{b^n = c}$$

Exponentiation is a mathematical operation, written as b^n, involving two numbers, the base b and the exponent or power n. When n is a positive integer, exponentiation corresponds to repeated multiplication of the base: that is, b^n is the product of multiplying n bases:
$b^n = b \times b \times \cdots \times b\ (n\ times)$
The exponent is usually shown as a superscript to the right of the base. In that case,
b^n is read
b raised (reɪzd) to the n-th power (ˈpaʊ ər),
b raised to the power of n,
the n-th power of b,
b to the nth,
or most briefly as b to the n.
E.g., 10^{-3} can be read as
ten raised to the minus third power, ten raised to the power of minus three, the minus third power of ten, ten to the minus three.
The expression $b^2 = b \cdot b$ is called "the square of b" or "b squared" because the area of a square with side-length b is b^2.
The expression $b^3 = b \cdot b \cdot b$ is called "the cube of b" or "b cubed" because the volume of a cube with side-length b is b^3.
With measurement units the order often changed, i.e.,
$(cm)^2$ or simply cm^2 is called centimeter squared or most often square centimeter
$(cm)^3$ or simply cm^3 is called centimeter cubed or most often cubic centimeter
Exponentiation with base 10 is used in scientific notation to denote large or small numbers. For instance, 299792458 m/s (the speed of light in vacuum, in meters per second) can be

written as 2.99792458×10^8 m/s and then approximated as 2.998×10^8 m/s. It is read as two point nine nine eight times ten to the 8 meter per second.

We often use the natural exponential function, f(x) = e^x or f(x) = exp(x) where e is the Euler's number (approximately≈ 2.718). e can be defined as

$$e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$$

see the meaning of limit in chapter 10, or e can be calculated by the summation:

$$\sum_{i=0}^{\infty}\frac{1}{i!} = e = \frac{1}{1}+\frac{1}{1}+\frac{1}{1\cdot 2}+\frac{1}{1\cdot 2\cdot 3}+\cdots$$

where i! (i factorial (fækˈtɔr i əl, -ˈtoʊr-) is the product of a given positive integer (ˈɪn tɪ dʒər) multiplied by all lesser positive integers: E.g., the quantity four factorial (4!) = 4 · 3 · 2 · 1 = 24, i.e.,

$$i! = \prod_{j=1}^{i} j$$

(The value of 0! is 1, according to the convention for an empty product.)

An nth root (rut, rʊt) of a number b is a number x such that $x^n = b$.

If b is a positive real number and n is a positive integer, then there is exactly one positive real solution to $x^n = b$. This solution is called the principal nth root of b. It is denoted

$$\sqrt[n]{b}$$

where √ is the radical (ˈræd ɪ kəl) sign or radical symbol or root symbol;

alternatively, the principal root may be written $b^{1/n}$. For example:

$$9^{\frac{1}{2}} = \sqrt[2]{9} = 3 \; and \; 8^{\frac{1}{3}} = \sqrt[3]{8} = 2$$

If n=2 it is called the square root

If n=3 it is called the cube root

9 Logarithm (ˈlɔ gəˌrɪð əm, -ˌrɪθ-, ˈlɒg ə-)

$$\log_b x = c$$

In mathematics, the logarithm is the inverse function to exponentiation. That means the logarithm of a given number x is the exponent to which another fixed number, the base b, must be raised, to produce that number x.

The logarithm of x to base b is denoted as $\log_b x$. The logarithm to base 10 (that is b = 10) is called the common logarithm and has many applications in science and engineering. Shortly is is denoted as lg (said ˈel dʒi).

The natural logarithm has the number e (that is b ≈ 2.718) as its base; its use is widespread in mathematics and physics, because of its simpler derivative. Shortly is is denoted as ln (said ˈel ɛn).

10 Limit (ˈlɪm ɪt)

$$\lim_{x\to c} f(x) = L$$

In mathematics, a limit (ˈlɪm ɪt) is the value that a function (or sequence) "approaches" as the input (or index) "approaches, →" some value.
In formulas, a limit of a function (f) is usually written as

$$\lim_{x \to c} f(x) = L$$

and is read as the limit of f (of) x as x approaches (əˈproʊtʃ) c equals L.
As for sequences, see e.g., Euler's number

$$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

To be read as e is the limit of the sequence of $\left(1 + \frac{1}{n}\right)^n$ as n approaches infinite (ˈɪn fə nɪt).
For finite (ˈfaɪ naɪt) values
if n=1, $\left(1 + \frac{1}{1}\right)^1 = 2$
if n=2, $\left(1 + \frac{1}{2}\right)^2 = 2.25$
if n=10, $\left(1 + \frac{1}{10}\right)^{10}$ =2.593742

if n=100, $\left(1 + \frac{1}{100}\right)^{100} = 2.704814$

if n=1000, $\left(1 + \frac{1}{1000}\right)^{1000} = 2.716924$

if n approaches, → ∞, $\left(1 + \frac{1}{\infty}\right)^{\infty} = e = 2.718\ 281\ 828\ 459\ 045\ 235\ 360\ 287\ 471\ 35\ldots$

11 Derivative (dɪˈrɪv ə tɪv)

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

The derivative of a function of a real variable measures the sensitivity to change of the function value (output value) with respect to a change in its argument (input value). Derivatives are a fundamental tool of calculus. For example, the derivative of the position of a moving object with respect to time is the object's velocity: this measures how quickly the position of the object changes when time advances.
Differentiation (ˌdɪf əˌrɛn ʃiˈeɪ ʃən) is the action of computing a derivative. The derivative of a function y = f(x) of a variable x is a measure of the rate at which the value y of the function changes with respect to the change of the variable x. It is called the derivative of f with respect to x. If x and y are real numbers, and if the graph of f is plotted against x, we can define the slope of this graph at each point as

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{\Delta y}{\Delta x}$$

called the ratio (quotient) of differences
If

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

exists, it is called the differential ratio (quotient), the derivative of f with respect to x.
It is often symbolized as

$$\frac{dy}{dx}$$

suggesting the ratio of two infinitesimal (infinitely small, differential) quantities.
The above expression is read as
the derivative of y with respect to x,
dy by dx (di waɪ baɪ di ɛks)
dy over dx.
The oral form "dy dx" is often used conversationally, although it may lead to confusion.

12 Integral (ˈɪn tɪ grəl, ɪnˈtɛg rəl)

$$\int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{f(x)\ dx}$$

The integral with respect to x of a real-valued function f(x) of a real variable x on the interval [a, b] is written as

$$\int_a^b f(x)\ dx$$

It is read as integral from a to b f of x (times) dx
The integral sign ∫ (elongated S) represents integration (as an abbreviation of an infinite summation of infinitesimal (infinitely small) products of f(x) dx. The symbol dx, called the differential of the variable x, indicates that the variable of integration is x. The function f(x) to be integrated is called the integrand (ˈɪn tɪˌgrænd) The symbol dx is separated from the integrand by a space (as shown). If a function has an integral, it is said to be integrable (ˈɪn tɪ grə bəl). The points a and b are called the limits of the integral. An integral where the limits are specified is called a definite (ˈdɛf ə nɪt) integral. The integral is said to be over the interval [a, b].
When the limits are omitted, as in

$$\int f(x)\ dx = F(x)$$

the integral is called an indefinite (ɪnˈdɛf ə nɪt) integral.
It is read as integral f of x dx.

It simply means, that

$$\frac{dF(x)}{dx} = f(x)$$

It is read as d F of x by dx is f of x